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SB-3452

M. A. / M. Sc. (Part-I) Examination

March / April – 2011

Mathematics : Paper - 404

(Complex Analysis)

(Old Course)

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. A. / M. Sc. (Part-1)**

Name of the Subject :
☛ **Mathematics - 404**

☛ Subject Code No. : **3 4 5 2** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) Answer **all** questions.

(3) Figures to the right indicate marks of the question.

(4) Follow usual notation.

1 (a) If $f(z)=u(x, y)+iv(x, y)$ is differentiable at $\alpha=x_0+iy_0$ then show that u and v satisfy the equations **6**

$$\frac{\partial u}{\partial x}(x_0, y_0)=\frac{\partial v}{\partial y}(x_0, y_0) \text{ and } \frac{\partial u}{\partial y}(x_0, y_0)=-\frac{\partial v}{\partial x}(x_0, y_0).$$

(b) Prove that if γ is a path, then the function of α defined by $\alpha \to \int_r \frac{dz}{z-\alpha}$ for an α not on the path is a continuous function of α. **4**

(c) Define Length of a curve. Find the length of a circle with center a and radius r. **4**

OR

1 (a) Let $\sum a_n z^n$ be the power series, then show that there exists a number $r > 0$ such that the series **6**

converges for every z such that $|z|<r$ and diverges for every z such that $|z|>r$ where $r=\frac{1}{\limsup.|a_n|^{\frac{1}{n}}}$.

(b) Let $w=f(z)$. Assume that f is differentiable at z and g is differentiable at w, then gOf is differentiable at z and $(gOf)'(z)=g'(f(z))f'(z)$. **4**

(c) If f and g are differentiable at z and $g(z)\neq 0$, then prove that : **4**

(i) $(f/g)'(z)=\frac{g(z)f'(z)-f(z)g'(z)}{[g(z)]^2}$

(ii) $(fg)'(z)=f'(z)g(z)+f(z)g'(z)$.

2 (a) Let R be a rectangle and let f be a function holomorphic on R. Then $\int_{\partial R} f=0$. **6**

(b) Let f be a continuous function on U. Let γ be a path in U then show that $|\int_\gamma f|\leq \|f\|_r L(r)$. **4**

(c) Expand $f(z)=\frac{1}{(z-1)(z-2)}$ for the following domain **4**

(i) $0<|z|<1$

(ii) $1<|z|<2$.

OR

2 (a) Let γ and η be two continuous paths in an open set U and they have the same beginning and end-point. Also they are close together. Let be f holomorphic on U then $\int_\gamma f=\int_\eta f$. **6**

(b) Evaluate $\int_\gamma \mathrm{Re}(z)dz$ where : **4**

(i) γ is the line segment from 0 to a+ib.

(ii) γ is the circle $|z|=R$.

(c) If f is bounded in some neighborhood of z_g, then one can define f(z) in a way such that the function is also analytic at z_g. **4**

3 (a) Show that $\int_0^{2\pi} \frac{ad\theta}{1+2a^2-\cos\theta} = \frac{\pi}{\sqrt{1+a^2}}$ **6**

(b) Prove that $e = e + e\sum_{i=1}^{\infty} \frac{1}{n!}(z-1)^n$. **4**

(c) Evaluate $\int_{\gamma} \bar{z}\, dz$ **4**

(i) $\gamma(t) = e^{\pi it}$; $0 \le t \le 1$

(ii) $\gamma(t) = e^{-\pi it}$; $0 \le t \le 1$.

OR

3 (a) State and prove local Cauchy formula. **6**

(b) Prove that $\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)^2} = \frac{2\pi a}{(a^2-b^2)^{\frac{3}{2}}}$ for 0<b<a. **4**

(c) Evaluate $\int_C \frac{\cos e^z}{z}$, where C is a unit circle. **4**

4 (a) State and prove cauchy residue formula. **6**

(b) If f(z) has simple pole at z_o and g(z) is holomorphic at z_o then prove that $\text{Res}_{z_o}(fg) = g(z_o)\text{Res}_{z_o} f$. **4**

(c) Let f be an entire function without zero's then prove that there exists an entire function h(z) such that $f(z) = e^{h(z)}$. **4**

OR

4 (a) Let u be continuous on the closed unit disc $\overline{D}$ and harmonic on the disc D. Let there exists an analytic function f on D such that u=Re(f), then show that $f(z)=\frac{1}{2\pi i}\int_C \frac{\xi+z}{\xi-z}\frac{u(\xi)}{\xi}d\xi+ik$, where C is the unit circle and k is a real constant. **6**

(b) Prove that $\pi\cot\pi z=\frac{1}{z}+2z\sum_{n=1}^{\infty}\frac{1}{z^2-n^2}$. **4**

(c) Show that f=u+iv is analytic if and only if $\frac{\partial\overline{f}}{\partial z}=0$. **4**

5 (a) Define Weierstrass function P(z) show that : **6**

$$p'(z)=-\frac{2}{z^3}+6S_4z+20S_6z^3+\ldots.$$

(b) Let f be an entire function of strict order $\leq p$. Let $V_f(R)$ be the number of zeros of f in the disc of radius R. Show that $V_f(R)\ll R^p$. **4**

(c) Let u be harmonic. Show that u^2 is subharmonic. **4**

OR

5 (a) Define a dirac sequence $\{K_n\}$. Let f be continuous and periodic then show that the sequence $\{K_n * f\}$ converges to f uniformly. **6**

(b) Let f be analytic on A and on its boundary C. Show that Green's theorem implies Cauchy's theorem. **4**

(c) Show that if u_1, u_2 are sub-harmonic and c_1, c_2 are positive numbers then $c_1u_1+c_2u_2$ is sub-harmonic. **4**